

0.2 Supplemental Exercises

1. Let $a > 0$ be a positive number. Compute

$$\sqrt{a + \sqrt{a + \sqrt{a + \dots}}}$$

2. Let $a > 0$ be a positive number. Compute

$$a + \frac{1}{a + \frac{1}{a + \dots}}$$

3. (i) Find $x > 0$ such that

$$x^{x^{x^{\dots}}} = 2.$$

- (ii) Find the largest possible value of $x > 0$ with such that there exists a number $b > 0$ with

$$x^{x^{x^{\dots}}} = b.$$

Also, what is the largest possible value of b ?

0.3 Solutions to Supplemental Exercises

Problem 1: Let $a > 0$ be a positive number. Compute

$$\sqrt{a + \sqrt{a + \sqrt{a + \dots}}}$$

Solution: If we know that the limit of $\sqrt{a + \sqrt{a + \sqrt{a + \dots}}}$ exists, and denote that limit by l , then it follows that l must satisfy

$$l = \sqrt{a + \sqrt{a + \sqrt{a + \dots}}} = \sqrt{a + l}, \quad (28)$$

which can be solved for l to obtain that

$$l = \frac{1 + \sqrt{1 + 4a}}{2}. \quad (29)$$