4.2 Supplemental Exercises

1. Show that the sequence

$$x_n \;=\; \left(\sum_{k=1}^{n} \frac{1}{k}\right) \;-\; \ln(n)$$

is convergent to a limit between 0 and 1.

Note: The limit of this sequence is $\gamma \approx 0.57721$, the Euler's constant.

2. Assume that an asset with spot price 50 paying dividends continuously at rate $q = 0.02$ has lognormal distribution with mean $\mu = 0.08$ and volatility $\sigma = 0.3$. Assume that the risk–free rates are constant and equal to $r = 0.05$.

(i) Find 95% and 99% confidence intervals for the spot price of the asset in 15 days, 1 month, 2 months, 6 months, and 1 year.

(ii) Find 95% and 99% risk–neutral confidence intervals for the spot price of the asset in 15 days, 1 month, 2 months, 6 months, and 1 year, i.e., assuming that the drift of the asset is equal to the risk–free rate.

3. If you play (American)[1]

roulette 100 times, betting \$100 on black each time, what is the probability of winning at least \$1000, and what is the probability of losing at least \$1000?

4. Use risk–neutral pricing to find the value of an option on a non–dividend–paying asset with lognormal distribution if the payoff of the option at maturity is equal to $\max((S(T))^\alpha - K, 0)$. Here, $\alpha > 0$ is a fixed constant.

5. Find a binomial tree parametrization for a risk–neutral probability (of going up or down) equal to $\frac{1}{2}$. In other words, find the up and down factors u and d such that

$$\begin{aligned} pu \;+\; (1-p)d \;&=\; e^{r\delta t}; \\ pu^2 \;+\; (1-p)d^2 \;&=\; e^{(2r+\sigma^2)\delta t}, \end{aligned}$$

if $p = \frac{1}{2}$.

[1] American roulette has 18 red slots, 18 black slots, and two green slots (corresponding to 0 and 00). European roulette, also called French roulette, has only one green slot corresponding to 0.