8.2 Supplemental Exercises

1. (i) If the current zero rate curve is

$$r_1(0,t) \;=\; 0.025 + \frac{1}{100}\exp\left(-\frac{t}{100}\right) + \frac{t}{100(t+1)},$$

   find the yield of a four year semiannual coupon bond with coupon rate 6%. Assume that interest is compounded continuously and that the face value of the bond is 100.

   (ii) If the zero rates have a parallel shift up by 10, 20, 50, 100, and 200 basis points, respectively, i.e., if the zero rate curve changes from $r_1(0,t)$ to $r_2(0,t) = r_1(0,t)+dr$, with $dr = \{0.001, 0.002, 0.005, 0.01, 0.02\}$, find out by how much does the yield of the bond increase in each case.

   Note: In general, a small parallel shift in the zero rate curves results in a shift of similar size and direction for the yield of most bonds (possibly with the exception of bonds with long maturity). This assumption will be tested for the bond considered here for parallel shifts ranging from small shifts (ten basis points) to large shifts (two percent).

2. Consider a six months at–the–money call on an underlying asset following a lognormal distribution with volatility 30% and paying dividends continuously at rate q. Assume that the interest rates are constant at 4%. Show that there is a unique positive value of q such that $\Delta(C) = 0.5$, and find that value using Newton's method. How does this value of q compare to $r + \frac{\sigma^2}{2}$?

3. The following prices of the Treasury instruments are given:

	Coupon Rate	Price
6 – Month T-bill	0	99.4565
12 – Month T-bill	0	98.6196
2 – Year T-bond	2	$101\frac{17.5}{32}$
3 – Year T-bond	4.5	$107\frac{18}{32}$
5 – Year T-bond	3.125	$102\frac{8}{32}$
10 – Year T-bond	4	$103\frac{8.5}{32}$

   The Treasury bonds pay semiannual coupons. Assume that interest is continuously compounded.

   (i) Use bootstrapping to obtain a zero rate curve from the prices of the 6–months and 12–months Treasury bills, and of the 2–year, 5–year and 10–year Treasury bonds;

(ii) Find the relative pricing error corresponding to the 3–year Treasury bond if the zero rate curve obtained at part (i) is used. In other words, price a 3–year semiannual coupon bond with 4.5 coupon rate and find its relative error to the price $107\frac{18}{32}$ of the 3–year Treasury bond.

8.3 Solutions to Supplemental Exercises

Problem 1: (i) If the current zero rate curve is

$$r_1(0,t) \;=\; 0.025 + \frac{1}{100}\exp\left(-\frac{t}{100}\right) + \frac{t}{100(t+1)},$$

find the yield of a four year semiannual coupon bond with coupon rate 6%. Assume that interest is compounded continuously and that the face value of the bond is 100.

(ii) If the zero rates have a parallel shift up by 10, 20, 50, 100, and 200 basis points, respectively, i.e., if the zero rate curve changes from $r_1(0,t)$ to $r_2(0,t) = r_1(0,t) + dr$, with $dr = \{0.001, 0.002, 0.005, 0.01, 0.02\}$, find out by how much does the yield of the bond increase in each case.

Solution: (i) The bond provides coupon payments equal to 3 every six months until 3.5 years from now, and a final cash flow of 103 in four years. By discounting this cash flows to the present using the zero rate curve $r_1(0,t)$, we find that the value of the bond is

$$\begin{aligned} B_1 &= \sum_{i=1}^{7} 3\,\exp\left(-r_1\left(0,\frac{i}{2}\right)\frac{i}{2}\right) \;+\; 103\,\exp(-4r_1(0,4)) \\ &= 106.1995. \end{aligned} \tag{8.36}$$

The yield of the bond is found by solving the formula for the price of the bond in terms of its yield, i.e, by solving

$$B_1 \;=\; \sum_{i=1}^{7} 3\,\exp\left(-y\,\frac{i}{2}\right) \;+\; 103\,\exp(-4y) \tag{8.37}$$

for y, where B_1 is given by (8.36), i.e., $B_1 = 106.1995$. Using Newton's method, we obtain that the yield of the bond is

$$y \;=\; 0.042511 \;=\; 4.2511\%.$$