ERRATA

A Primer for the Mathematics of Financial Engineering

First Edition, 2008

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Corrections – Chapter 0

- **Page 16**, Section 0.5, Exercise 4, part (iii): the sequence should be denoted $(x_n)_{n\geq 1}$ and not $(x_n)_{n\geq 0}$

Corrections – Chapter 1

- **Page 30**, Section 1.6: on line 6 from the top, $e_i(j) = 1$ should be $e_i(i) = 1$.
- **Page 39**, Section 1.10: formula $V(0) = e^{-r(T-t)}V(T)$ should be $V(0) = e^{-rT}V(T)$.
- **Page 42**, Section 1.12, Exercise 8: The sentence "When scaled appropriately, the derivative of f(y) with respect to y give the duration and convexity of the bond" should be "When scaled appropriately, $f'(y)$ and $f''(y)$ give the duration and convexity of the bond, respectively"

Corrections – Chapter 2

- **Page 43**, Section 2.4: Formula

$$I_8^M \;=\; h\sum_{i=1}^{8} f(x_i) \;=\; \frac{1}{4}\sum_{i=1}^{8}\frac{1}{(1+(i-1/2)/4)^2} \;=\; 0.24943374$$

should be

$$I_8^M \;=\; h\sum_{i=1}^{8} f(x_i) \;=\; \frac{1}{4}\sum_{i=1}^{8}\frac{1}{(2+(i-1/2)/4)^2} \;=\; 0.24943374.$$

In the expression for I_8^S,

$$\frac{1}{24}\left(\frac{1}{4} + 2\sum_{i=1}^{7}\frac{1}{(2+i/4)^2} + \frac{1}{16} \;+\; 4\sum_{i=1}^{8}\frac{1}{(1+(i-1/2)/4)^2}\right)$$

should read

$$\frac{1}{24}\left(\frac{1}{4} + 2\sum_{i=1}^{7}\frac{1}{(2+i/4)^2} + \frac{1}{16} \;+\; 4\sum_{i=1}^{8}\frac{1}{(2+(i-1/2)/4)^2}\right)$$

- **Page 55**, Section 2.4 : On line 9 from the bottom, I_n^s should be I_8^s
- **Page 57**, Section 2.5: the term $f''(\xi_{i,T})$ in formula (2.28) should be $f''(\xi_{i,M})$; the term $f''(\xi_{i,M})$ in formula (2.29) should be $f''(\xi_{i,T})$; the terms $f''(\xi_{i,T})$ in the last formula on page 57 should be $f''(\xi_{i,M})$
- **Page 57**, Section 2.5: Line 3 from the bottom should be

$$|I - I_n^M| \;=\; \left|\;\sum_{i=1}^{n}\left(\int_{a_{i-1}}^{a_i} f(x)\;dx \;-\; \int_{a_{i-1}}^{a_i} c_i(x)\;dx\right)\;\right|$$

- **Page 58**, Section 2.5: the inequality $|f''(\xi_{i,T})| \leq \max_{a\leq x\leq b} |f''(x)|$ should be $|f''(\xi_{i,M})| \leq \max_{a\leq x\leq b} |f''(x)|$
- **Page 62**, Section 2.5.2: the tolerance is $tol = 5 \cdot 10^{-7}$, not $tol = 0.5 \cdot 10^{-7}$
- **Page 77**, Section 2.8: The input for the code is $y = 0.065$, not $y = 0.06$
- **Page 78**, Section 2.10, Exercise 2, part(i): formula $\int_0^1 x^{\alpha-1}\, e^{-x}\, dx \;=\; \lim_{t\to 0} \int_t^1 x^{\alpha-1}\, e^{-x}\, dx$ should be
$$\int_0^1 x^{\alpha-1}\; e^{-x}\; dx \;=\; \lim_{t\searrow 0} \int_t^1 x^{\alpha-1}\; e^{-x}\; dx$$
- **Page 79**, Section 2.10, Exercise 6: Let $h(x)$ be a continuous function such that $\int_{-\infty}^{\infty} |xh(x)|dx$ exists. Define $g(t)$ by
$$g(t) \;=\; \int_t^{\infty} (x-t)h(x)\; dx,$$
and show that
$$g''(t) \;=\; h(t).$$
- **Page 79**, Section 2.10, Exercise 8: Instead of "... the continuously compounded instantaneous interest curve..." should be "... the continuously compounded instantaneous interest rate curve..."

Corrections – Chapter 3

- **Page 82**, Section 3.1: On the third line from the bottom, the function should be $X : S \to \mathbb{R}$, instead of $X : S \to [0,1]$.
- **Page 85**, Section 3.2: In the statement of Lemma 3.4, the inequality $\int_{\mathbb{R}} |h(x)f(x)dx < \infty$ should be $\int_{\mathbb{R}} |h(x)|f(x)dx < \infty$.
- **Page 97**, Section 3.6: In the first paragraph, the sentence "They change are called "the Greeks" of the portfolio" should read "These changes are called "the Greeks" of the portfolio".
- **Page 104**, Section 3.6.2: Equation (3.93) and the text thereafter should read
$$P_{BS}(\sigma_{imp,P}) = P \quad \text{and} \quad C_{BS}(\sigma_{imp,C}) = C,$$
where we denoted $P_{BS}(S, K, T, \sigma_{imp,P}, r, q)$ and $C_{BS}(S, K, T, \sigma_{imp,C}, r, q)$, respectively, by $P_{BS}(\sigma_{imp,P})$ and $C_{BS}(\sigma_{imp,C})$.
- **Page 110**, Section 3.8: the value of the put is $P = 2.343020$, not $P = 2.343022$
- **Page 111**, Section 3.10, Exercise 4 (iv): formula $P(X \;\geq\; t) \;=\; \int_t^{\infty} f(x)\; dx \;=\; e^{\alpha t}$ should be
$$P(X \;\geq\; t) \;=\; \int_t^{\infty} f(x)\; dx \;=\; e^{-\alpha t}$$

Corrections – Chapter 4

- **Page 123**, Section 4.3: In Theorem 4.1, the equality should be
$$(f_1 * f_2)(x) \;=\; \int_{-\infty}^{\infty} f_1(z) f_2(x-z)\; dz \;=\; \int_{-\infty}^{\infty} f_1(x-z) f_2(z)\; dz$$
- **Page 126**, Section 4.3: The last sentence of Theorem 4.3 should be "Then $Y_1 Y_2$ is a lognormal variable with parameters $\mu_1 + \mu_2$ and $\sqrt{\sigma_1^2 + \sigma_2^2}$". Also, the last sentence in the proof of Theorem 4.3 should be "which shows that $Y_1 Y_2$ is a lognormal random variable...", instead of "which shows that $\ln(Y_1\, Y_2)$ is a lognormal random variable..."
- **Page 129**, Section 4.5: Middle of the page. Replace
"If $x = 1$, the partial sum $\sum_{k=0}^{n} x^k$ is equal to n..." by
"If $x = 1$, the partial sum $\sum_{k=0}^{n} x^k$ is equal to $n+1$...".

- **Page 136**, Section 4.8: Formula (4.59) should be

$$d_{2,\mu} \ = \ \frac{\ln\left(\frac{S(0)}{K}\right) + \left(\mu - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

  Formula (4.60) should be:

$$P(S(T) \ > \ K) \ = \ N(d_{2,\mu}) \ = \ N\left(\frac{\ln\left(\frac{S(0)}{K}\right) + \left(\mu - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$$

- **Page 137**, Section 4.8: The first formula on the page, instead of

$$d_2 \ = \ \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

  should read

$$d_2 \ = \ \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}.$$

Corrections – Chapter 6

- **Page 179**, Section 5.1: Formula (6.10) should be

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2}f''(a) + \cdots + \frac{(x-a)^n}{n!}f^n(a) + O\left((x-a)^{n+1}\right)$$

Corrections – Chapter 7

- **Page 210**, Section 7.3, Theorem 7.1 should read
  Theorem 7.1. Let (x_0, y_0) be a critical point for the function $f(x, y)$ and assume that all the second order partial derivatives of $f(x, y)$ exist and are continuous.
  If the matrix $D^2f(x_0, y_0)$ is positive definite, i.e., if both eigenvalues of the matrix $D^2f(x_0, y_0)$ are strictly positive, then the critical point (x_0, y_0) is a local minimum point;
  If the matrix $D^2f(x_0, y_0)$ is negative definite, i.e., if both eigenvalues of the matrix $D^2f(x_0, y_0)$ are strictly negative, then the critical point (x_0, y_0) is a local maximum point;
  If the two eigenvalues of the matrix $D^2f(x_0, y_0)$ are nonzero and of opposite signs, then the critical point (x_0, y_0) is a saddle point, and it is not an extremum point;
  In any other case, anything could happen, i.e., the critical point (x_0, y_0) could be a local maximum point, a local minimum point, or a saddle point.
- **Page 215**, Section 7.3, Theorem 7.3 should read
  Theorem 7.3. Let $f : U \to \mathbb{R}$ be a twice differentiable function, with continuous second order partial derivatives, and let x_0 be a critical point for the function $f(x)$. Then, the Hessian matrix of f evaluated at x_0, i.e., $D^2f(x_0)$, is a symmetric matrix and all the eigenvalues of $D^2f(x_0)$ are real numbers.
  If all the eigenvalues of $D^2f(x_0)$ are strictly greater than 0 (i.e., if $D^2f(x_0)$ is a symmetric positive definite matrix), then x_0 is a local minimum point.
  If all the eigenvalues of $D^2f(x_0)$ are strictly less than 0 (i.e., if $D^2f(x_0)$ is a symmetric negative definite matrix), then x_0 is a local maximum point.
  If the matrix $D^2f(x_0)$ has both positive and negative eigenvalues, then the point x_0 is a saddle point, i.e., x_0 is neither a local minimum, nor a local maximum.

- **IMPORTANT: Page 222**, Section 7.7: Formula (7.36) for b should be

$$b = \left(\frac{r-q}{\sigma^2} + \frac{1}{2}\right)^2 + \frac{2q}{\sigma^2}$$

- **Page 231**, Section 7.11, Exercise 1: show that $\lim_{S \searrow 0} \Gamma(S) = 0$, not $\lim_{S \to 0} \Gamma(S) = 0$
- **Page 231**, Section 7.11, Exercise 4: the formula for b should be

$$b = \left(\frac{r-q}{\sigma^2} + \frac{1}{2}\right)^2 + \frac{2q}{\sigma^2}$$

- **Page 233**, Section 7.11, Exercise 6: the first sentence on page 233 should read
Thus, the value $V(S, I, t)$ of an Asian option depends not only on the spot price S of the underlying asset and on the time t, but also on the following random variable:

$$I(t) = \int_0^t S(\tau)\ d\tau$$

- **Page 233**, Section 7.11, Exercise 7: "Fill in the Black–Scholes values of the OTM put options..." should be "Fill in the Black–Scholes values of the ITM put options..." After the table, the sentence "For which of these options is the intrinsic value $\max(K-S,0)$ larger than the price of the option" should be "For which of these options is the intrinsic value $\max(K-S,0)$ larger than the Black–Scholes value of the option"
- **Page 233**, Section 7.11, Exercise 8: "Show that the premium of the price of a European call option..." should be "Show that the premium of the Black–Scholes value of a European call option..."
- **Page 233**, Section 7.11, Exercise 9: "Use formula (7.41) to price a six months down–and–out call..." should be "Use formula (7.41) to find the value of a six months down–and–out call..."
- **Page 234**, Section 7.11, Exercise 10: the Hint should read "Use formula (7.41) to show that the value of the down–and–out call is

$$V(S) = C(S) - \left(BN(d_1) - \frac{SK}{B}e^{-rT}\ N(d_2)\right),$$

where $C(S)$ is the value at time 0 of the plain vanilla call with strike K..."

Corrections – Chapter 8

- **Page 236**, Section 8.1: formula (8.6) should be

$$\nabla_x\ F(x,\lambda) = \left(\frac{\partial F}{\partial x_1}(x,\lambda)\ \ldots\ \frac{\partial F}{\partial x_n}(x,\lambda)\right) = \nabla f(x) + \lambda^t(\nabla g(x))$$

- **Page 236**, Section 8.1: formula (8.7) should be

$$\nabla_\lambda\ F(x,\lambda) = \left(\frac{\partial F}{\partial \lambda_1}(x,\lambda)\ \ldots\ \frac{\partial F}{\partial \lambda_m}(x,\lambda)\right) = (g(x))^t$$

- **Page 237**, Section 8.1: The first formula on the page,

$$\nabla f(x) = \left(\frac{\partial f}{\partial x_1}(x,\lambda)\ \ldots\ \frac{\partial f}{\partial x_n}(x,\lambda)\right);$$

should read

$$\nabla f(x) = \left(\frac{\partial f}{\partial x_1}(x)\ \ldots\ \frac{\partial f}{\partial x_n}(x)\right);$$

- **Page 237**, Section 8.1: formula (8.8) should be

$$\nabla_{(x,\lambda)}\ F(x,\lambda) = \left(\ \nabla f(x) + \lambda^t(\nabla g(x))\ \ (g(x))^t\ \right)$$

- **Page 239**, Section 8.1: formula (8.11) should be

$$D^2F_0(x_0) \;=\; \begin{pmatrix} \frac{\partial^2 F_0}{\partial x_1^2}(x_0) & \frac{\partial^2 F_0}{\partial x_2 \partial x_1}(x_0) & \dots & \frac{\partial^2 F_0}{\partial x_n \partial x_1}(x_0) \\ \frac{\partial^2 F_0}{\partial x_1 \partial x_2}(x_0) & \frac{\partial^2 F_0}{\partial x_2^2}(x_0) & \dots & \frac{\partial^2 F_0}{\partial x_n \partial x_2}(x_0) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 F_0}{\partial x_1 \partial x_n}(x_0) & \frac{\partial^2 F_0}{\partial x_2 \partial x_n}(x_0) & \dots & \frac{\partial^2 F_0}{\partial x_n^2}(x_0) \end{pmatrix}$$

- **Page 239**, Section 8.1: formula (8.12) should be

$$q(v) \;=\; v^t\ D^2F_0(x_0)\ v \;=\; \sum_{1 \le i,j \le n} \frac{\partial^2 F_0}{\partial x_i \partial x_j}(x_0) v_i v_j$$

- **Page 240**, Section 8.1: Third formula from the top should be

$$q(v) \;=\; v^t\ D^2F_0(x_0)\ v \;=\; v_1^2 + 2v_2^2 + 3v_3^2 - 2v_1v_2 + 4v_2v_3$$

- **Page 240**, Section 8.1: The footnote should read
  "For example, the quadratic form q_{red} from (8.15) is positive semidefinite since

$$q_{red}(v_{red}) \;=\; (v_2 + 2v_3)^2 + 3v_3^2 \;\ge\; 0,\ \forall\ (v_2, v_3) \in \mathbb{R}^2,$$

  and $q_{red}(1,1) = 12 > 0$."
- **Page 241**, Section 8.1: In Theorem 8.2., the formula for the Lagrangian function should be "$F(x,\lambda) = f(x) + \lambda^t g(x)$", not "$F(x,\lambda) = f(x) + \lambda_0^t g(x)$.
- **IMPORTANT: Page 241**, Section 8.1: Step 3.2 should read as follows:

  Step 3.2: Compute $q_{red}(v_{red})$ by restricting $q(v)$ to the vectors v satisfying the condition $\nabla g(x_0)\ v = 0$.

- **Page 248**, Section 8.2.1: the number of iterations in the bisection method example is 35, not 33
- **Page 252**, Section 8.2.2: The sentence "For example, if $x_0 = 0.001$, then $x_1 = 350.0269$" should read "For example, if $x_0 = 0.01$, then $x_1 = 350.0269$"
- **Page 260**, Section 8.3: The sentence "For the initial guess $x_0 = (2\ 2\ 2)^t$, the solution $x^* = (-1\ 3\ 1)^t$ is found after 58 iterations" should read "For the initial guess $x_0 = (2\ 2\ 2)^t$, the solution $x^* = (-1\ 3\ 1)^t$ is found after 65 iterations"
- **Page 263**, Section 8.4: In the first formula, $\nabla(x,\lambda)F(x,\lambda)$ should be $\nabla(w,\lambda)F(w,\lambda)$
- **Page 264**, Section 8.4: On row four from the top of the page, the equality $\nabla g(x_0)\ v = 0$ should be $\nabla g(w_0)\ v = 0$
- **Page 264**, Section 8.4: The footnote at the bottom of the page should read "Note that $D^2F_0(w)$ is equal to twice the covariance matrix of the rates of return $(R_i)_{i=1:4}$, which is a positive definite matrix"
- **Page 271**, Section 8.7: formula (8.78) should be

$$109 \;=\; 2\ \sum_{i=1}^{9} e^{-0.5i\ r(0,0.5i)} \;+\; 102\ e^{-5r(0,5)}$$

- **Page 277**, Section 8.9, Exercise 8: The first sentence should be "Use bootstrapping to obtain a continuously compounded zero rate curve given the prices..."
  The last formula on Page 277 should be

$$r\left(0, \frac{7}{6}\right) \;=\; \frac{3r(0,1) + 4r\left(0, \frac{5}{3}\right)}{4}.$$