

**ELEMENTS OF
STOCHASTIC PROCESSES:
A COMPUTATIONAL
APPROACH**

C. Douglas Howard
Baruch College
City University of New York

FE PRESS
New York

**ELEMENTS OF STOCHASTIC PROCESSES:
A COMPUTATIONAL APPROACH**

CONTENTS

Preface	v
Chapter I. Essentials of Probability	1
§1. Probability Spaces	1
§2. Random Variables and Expectation	7
§3. Moments, Variance, Covariance, and Correlation	12
§4. Jointly Distributed Random Variables	15
§5. IID Sequences of Random Variables	19
§6. The Unit Interval $\Omega = (0, 1)$ as Outcome Space	21
§7. Constructing Stochastic Processes on Ω	25
Chapter II. Some Fundamental Limit Theorems	31
§8. Two Types of Convergence	31
§9. The Weak Law of Large Numbers	35
§10. The Strong Law of Large Numbers	38
§11. The Central Limit Theorem	43
§12. The Monotone Convergence Theorem	48
§13*. Two More Limit Theorems for Expectation	51
§14*. Recovering the Moments of X from $M_X(\theta)$	52
Chapter III. Markov Chains with Finitely Many States	55
§15. The Two-State System	55
§16. Markov Chains with More than Two States	60
§17. Irreducible and Ergodic Markov Chains	64
§18. Example: An Interacting Particle System	75
§19. A Strong Law for Ergodic Markov Chains	79
§20. Communication Classes and Their Properties	82
§21. When the Markov Chain is Periodic	88
§22. Ergodic Decomposition	95

Chapter IV. Random Walks	101
§23. Preliminaries	101
§24. Random Walks on \mathbb{Z}	102
§25. Random Walks in Two Dimensions	116
§26. Random Walks in Three Dimensions	120
§27. Stirling's Formula	125
Chapter V. Arrival Processes and Poisson Point Processes	129
§28. A Motivating Example	129
§29. The Homogeneous Poisson Process in One Dimension	132
§30. The Inhomogeneous Poisson Process	140
§31. When Inter-Arrival Times are IID but not Exponential	148
Chapter VI. Brownian Motion	155
§32. Introduction	155
§33. The "Bumpiness" of Brownian Motion	158
§34. Recurrence	166
§35*. The Lévy Construction of Brownian Motion	169
Chapter VII. A Glimpse of Stochastic Calculus	179
§36. Introduction	179
§37. Stochastic Differential and Integral Equations	180
§38. Itô's Formula for Brownian Motion	183
§39. Application: The Black-Scholes PDE and Formula	187
§40. The Ornstein-Uhlenbeck Process	194
§41. Application: Modeling the Index of Consumer Sentiment	204
Appendix I. Some Common Distributions	211
§42. Some Common Discrete Distributions	211
§43. Some Common Continuous Distributions	216
Appendix II. Some Real Analysis	219
§44. Completeness and Some Consequences	219
§45. Basics of Estimation	225
Bibliography	231
Index	233